MOIST BAROCLINIC INSTABILITY IN THREE DIMENSIONS

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1. INTRODUCTION

The effects of condensation of water vapour and consequent release of latent heat within baroclinic waves was represented by Emanuel *et al.*, 1987 (hereafter referred to as EFT) with reduced potential vorticity in the updraft region of such waves, in the assumption that a slantwise convective adjustment occurred. The study of two- dimensional models incorporating this assumption revealed a faster growth rate and a contraction of the width of the updraft region. Those features were also obtained in other 2-D models with explicit representation of convection and various approximations for the meridional advection of thermodynamic quantities (e.g. Fantini, 1993). All the results concerning linear non-adiabatic baroclinic instability need to be confirmed in more realistic three-dimensional geometry. In particular, by showing that the most unstable normal modes are two-dimensional one could give confidence to the results of two-dimensional models. In this work we use a three dimensional quasi geostrophic model, designed specifically for this purpose, to find the normal modes of the Eady problem when condensation effects are represented as in EFT. The choice of that parameterization for the release of latent heat is motivated in the first place by the desire to extend to three dimensions the known results. It has also the advantage of not introducing any meridional scale of its own, as would be the case for any other representation taking into account the variation of saturation vapour pressure with latitude related to environmental baroclinicity.

The present model is quasi geostrophic rather than semi geostrophic as the models used in EFT were. It was argued in Fantini, 1990 that the appearance of Ertel's potential vorticity as a coefficient in the pseudo potential vorticity equation is an artifact of the semi geostrophic approximation, and both the primitive equations and the quasi geostrophic equations are sensitive to 'vertical' static stability. We hope we will be able to clarify this one, as well as other problems related to the representation of the meridional and vertical variation of environmental parameters, by performing experiments with a hierarchy of models, of which the quasi geostrophic is but the first step, and which will include a 'geostrophic momentum' extension of the present one, and a full primitive equation non hydrostatic cloud model adapted to the study of moist baroclinic waves. The present work will only be concerned with the presentation of baroclinic instability in an environment of small stability to moist slantwise convection in the quasi geostrophic framework. A linearized perturbation approach about an Eady base state is assumed, but some preliminary results in the non linear regime will also be included.

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2. QUASI GEOSTROPHIC MODEL

The parameterization of latent heating effects introduced by EFT, translated in the quasi geostrophic framework, consists of using a 'dry' static stability N_d for all descending motion, and a much smaller 'moist' static stability N_m for ascending motions. The value of N_m is determined by the assumption that a slantwise convective adjustment has occurred and reduced the thermodynamic profile to a state of neutrality to moist slantwise convection. This condition is expressed quantitatively by $q_e = 0$, where q_e is the equivalent potential vorticity. The observational basis for this assumption were presented by Emanuel, 1988. The numerical formulation of this parameterization requires the knowledge of vertical velocity w at each grid point and at each time step. For this purpose the quasi geostrophic pseudo potential vorticity equation is not integrated in the usual form containing just the geopotential/ streamfunction ϕ, but is instead expressed, in non dimensional form, as

$$(\partial_t + \phi_x \partial_y - \phi_y \partial_x)\, \nabla_H^2 \phi = w_z \tag{1}$$

where

$$\nabla_H^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The thermodynamic equation is

$$(\partial_t + \phi_x \partial_y - \phi_y \partial_x)\, \phi_z + n^2 w = 0 \tag{2}$$

where

$$n = \begin{cases} 1 & \text{where } w < 0 \\ \dfrac{N_m}{N_d} & \text{where } w > 0 \end{cases}$$

From (1) and (2) we obtain the diagnostic equation for w ('ω equation')

$$\nabla_H^2(n^2 w) + w_{zz} = 2\left(\phi_{xz}\partial_y - \phi_{yz}\partial_x\right)\nabla_H^2\phi \qquad (3)$$

Equation (1) is integrated forward in time with a leapfrog scheme, and (3) is solved by relaxation at each time step in order to obtain w, and consequently the n^2 necessary for the next time step. The relaxation technique has to be used because of the coupling of n^2 and w in the horizontal laplacian term, which requires an initial guess and successive adjustments toward the correct solution. On the other hand, the inversion of $\nabla_H^2\phi$ to obtain the geopotential is performed by double Fourier transform, since the domain chosen is doubly periodic.

The scaling quantities are:

$$\text{length}: \qquad L = \frac{N_d H}{f_o}$$

$$\phi: \qquad f_o U L$$

$$w: \qquad \frac{UH}{L}\frac{U}{f_o L}$$

where as usual H is the height of the model, U the zonal wind increase between top and bottom boundaries and f_o the uniform Coriolis parameter. Extension of this model to β-plane(t) dynamics is under way.

The condition $q_e = 0$, representative of the state of neutrality to moist slantwise convection, gives the non dimensional value $n^2 = r = .05$ in the updraft, while $r = 1$ recovers the dry Eady model.

Perturbing the geopotential around an Eady base state

$$\phi = -(u_o + z)y + \phi'$$

gives the equations as they are actually integrated in the model:

$$(\partial_t + z\partial_x)\nabla_H^2\phi' + \lambda\left(\phi'_x\partial_y - \phi'_y\partial_x\right)\nabla_H^2\phi'$$

$$= w_z + \lambda\nu\nabla_H^2\left(\nabla_H^2\phi\right) \qquad (4)$$

$$\nabla_H^2(n^2 w) + w_{zz} =$$

$$2\nabla_H^2\phi' + 2\lambda\left(\phi'_{xz}\partial_y - \phi'_{yz}\partial_x\right)\nabla_H^2\phi' \qquad (5)$$

where we have introduced a flag λ to mark the non-linear terms and added a laplacian diffusion, with coefficient $\nu = .01$, in the prognostic equation to control numerical noise in the non-linear integrations. Finally the vertical discretization is of the Lorenz type, with ϕ known at levels intermediate between w levels. The vertical velocity w is set equal to zero at top and bottom boundaries. All experiments presented here have been run with 11 vertical levels and 32 grid points in either x and y directions.

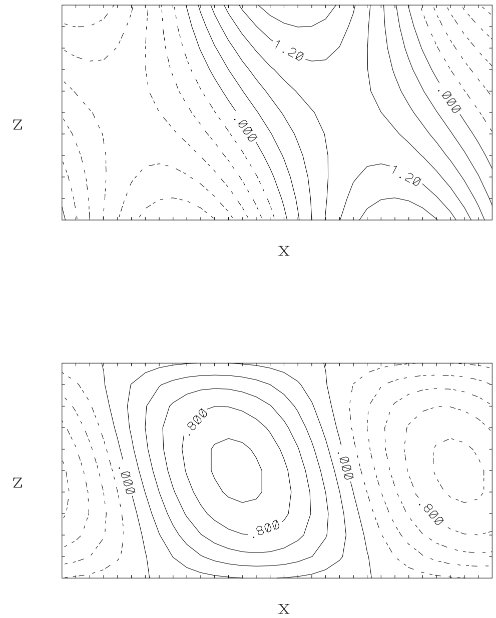

Fig.1 – Geopotential (top) and vertical velocity (bottom) for dry Eady wave of non dimensional zonal length 3.6, infinite in y

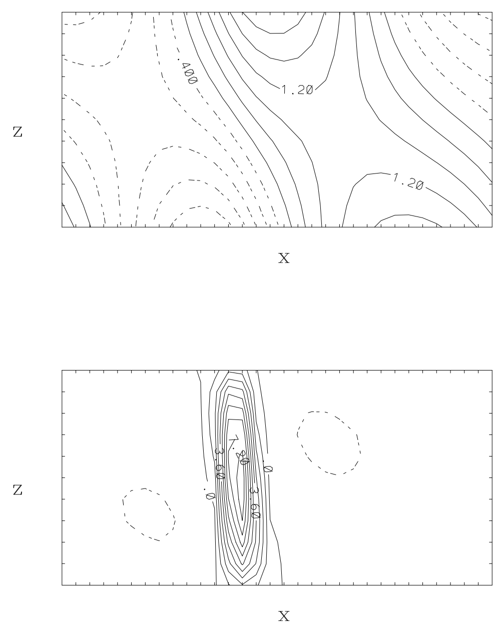

Fig.2 – Same as Fig.1 except moist wave, r=.05

3. LINEAR RESULTS

We begin by showing in Fig.1 the $x - z$ cross section of geopotential ϕ and vertical velocity w for a dry Eady wave ($r = 1$) of non dimensional length $X = 3.6$ in the zonal direction and infinite meridional wavelength. This is near the most unstable dry Eady wave. Fig.2 shows the same fields for this wave in an environment having $r = .05$ and displays the features already known from the previously mentioned works. We also show in Fig.3 the growth rate σ versus wavelength X for dry and moist Eady waves of infinite meridional extent as obtained in the present model.

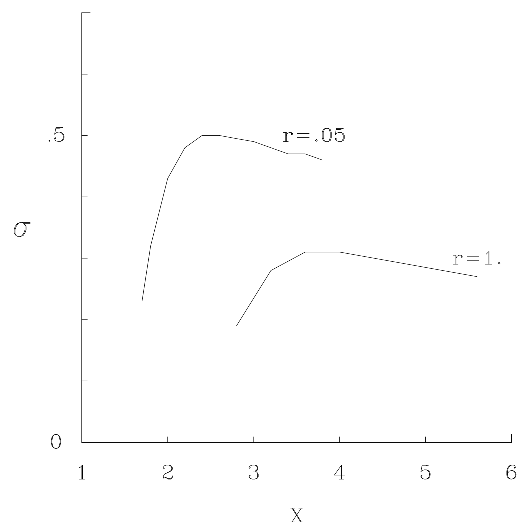

Fig.3 – Growth rate vs zonal wavelength for dry (r=1) and moist (r=.05) Eady waves of infinite meridional wavelength, as obtained in the present model

The results shown up to now were obtained by initializing the time run with a dry Eady normal mode of infinite meridional wavelength. If we use an initial condition with a nonzero meridional wavenumber, the growth rate remains smaller than that of the two-dimensional wave and no contraction of the meridional scale is observed. Fig.s 4 and 5 present the geopotential and vertical velocity, respectively, at the first internal level of the model (non dimensional height .045), for one such case with zonal wavelength $X = 2.4$, which is the fastest growing wavelength in the moist environment, and meridional wavelength $Y = 5.0$.

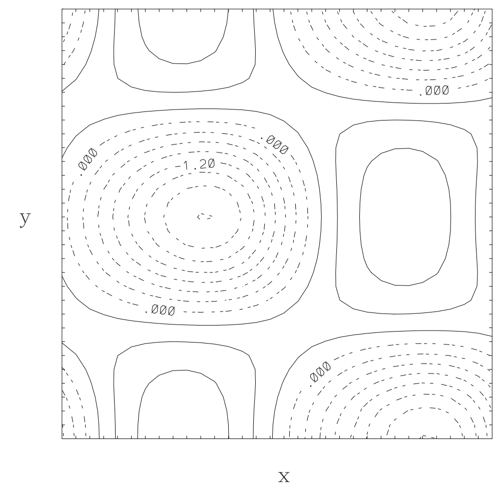

Fig.4 – Geopotential ϕ at the lowest internal level for linear normal mode of zonal length $X = 2.4$ and meridional length $Y = 5.0$ non dimensional units in the $r = .05$ environment

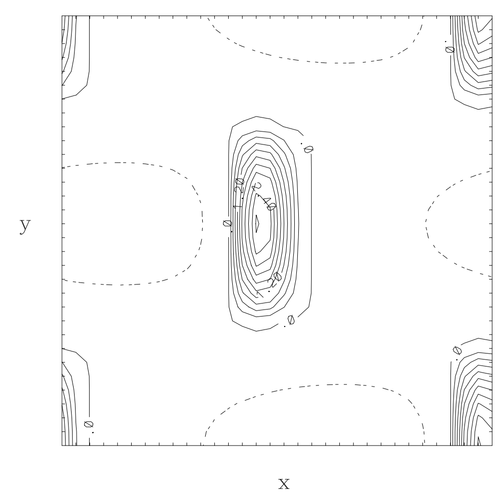

Fig.5 – Same as Fig.4 but for vertical velocity w

As a further check on the 'normal mode' nature of those solutions, and their ranking in terms of growth rate we have performed a series of experiments started from random initial conditions and show in Fig.6 the time evolution of the w field at the first internal level for one of those runs, which shows very clearly how a nonzero meridional wavenumber is firstly apparent, because of accidental meridional structure of the initial condition, but slowly the purely two-dimensional wave takes over, thanks to the marginally higher growth rate.

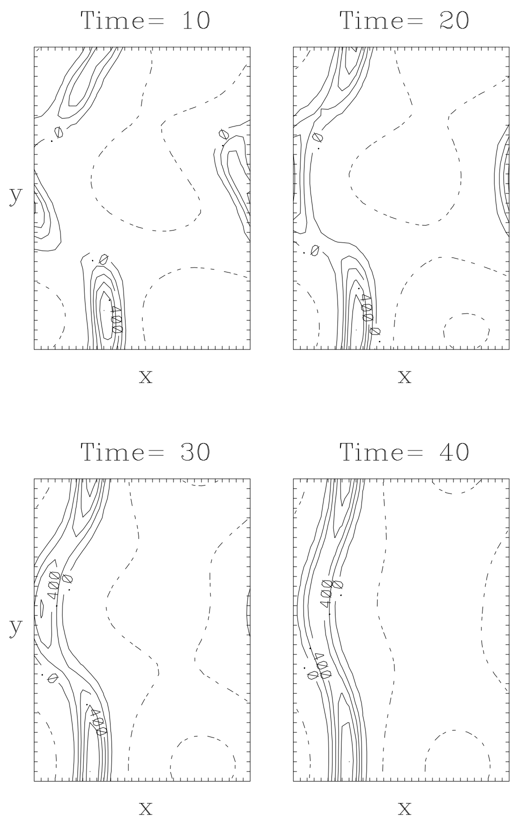

Fig.6 – Time evolution of the vertical velocity field at the lowest internal level for a run which started from random initial condition. $X = 2.4, Y = 5.0$. Maximum amplitude is normalized to 1. Isoline spacing for negative (dashed) values is 1/3 of the spacing for positive values.

4. NON LINEAR EVOLUTION

The result shown so far constitute fair evidence that the most unstable normal modes of moist baroclinic instability are two-dimensional, having parameterized the release of latent heat as in EFT. We know that a more realistic representation of saturation vapour pressure would introduce a 'physical' meridional scale, just like the consideration of a zonal jet would influence the meridional scale of baroclinic waves, and we plan to tackle those problems with more appropriate models in the future. The main question we wanted to answer here was whether the scale contraction induced by condensation of vater vapour in the zonal direction would appear in the meridional direction as well. This does not seem to be the case as far as the linear modes are concerned. On the other hand it is well known that convection in its pure form is spatially isotropic and the only term in the linear baroclinic equations which can break this symmetry is the advection by the zonal wind. It is then natural to expect that in the non linear phase of evolution, when the meridional self-advection term becomes $o(1)$, a meridional scale contraction may occur.

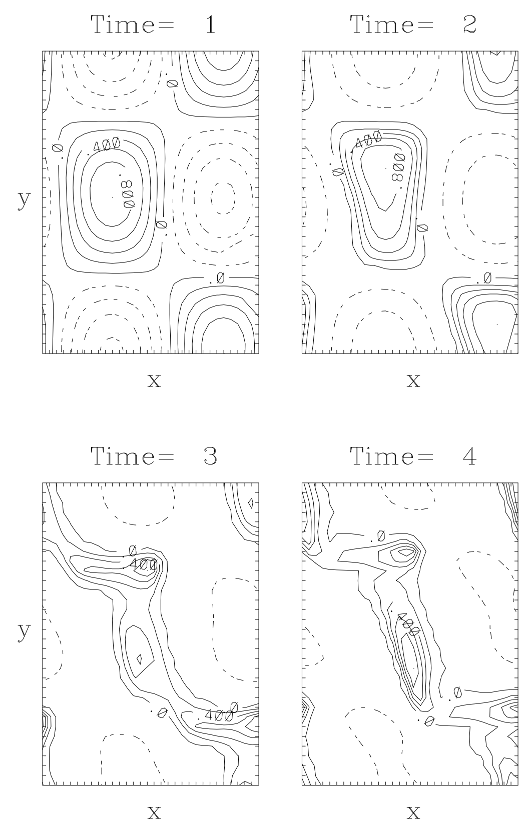

Fig.7 – Time evolution of w in a non-linear experiment for $X = 2.4, Y = 5.0$ as in Fig.5. The initial condition is a dry Eady wave.

Fig.7 shows the time evolution of vertical velocity for the same wave of Fig.5 when the non-linear terms are active ($\lambda = 1$ in Eq.s (4) and (5)). The expected scale contraction takes place disuniformly in space. The updraft appears split in a sequence of patches which are alternately elongated in the meridional and in the zonal direction and are aligned with the eastern and northern boundaries of the low pressure areas (not shown). A detailed study of this evolution and its meteorological implications will be presented elsewhere.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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